5–9 Factoring Pattern for $ax^2 + bx + c$

Objective: To factor general quadratic trinomials with integral coefficients.

Patterns

Factoring pattern for $ax^2 + bx + c$: $(px + r)(qx + s)$.

Example 1   Factor $2x^2 - 3x - 9$.

Solution

Clue 1   Because the trinomial has a negative constant term, one of r and s will be negative and the other will be positive.

Clue 2   You can list the possible factors of the quadratic term, $2x^2$, and the possible factors of the constant term, -9.

Factors of $2x^2$	Factors of -9	
$2x, x$	$1, -9$	$-1, 9$
	$3, -3$	$-3, 3$
	$9, -1$	$-9, 1$

Make a chart to test the possibilities to see which produces the correct linear term, $-3x$.

Since $(2x + 3)(x - 3)$ gives the correct linear term,
$2x^2 - 3x - 9 = (2x + 3)(x - 3)$.

Possible factors	Linear Term
$(2x + 1)(x - 9)$	$(-18 + 1)x = -17x$
$(2x + 3)(x - 3)$	$(-6 + 3)x = -3x$ ←
$(2x + 9)(x - 1)$	$(-2 + 9)x = 7x$
$(2x - 1)(x + 9)$	$(18 - 1)x = 17x$
$(2x - 3)(x + 3)$	$(6 - 3)x = 3x$
$(2x - 9)(x + 1)$	$(2 - 9)x = -7x$

Example 2   Factor $10x^2 - 11x + 3$.

Solution

Clue 1   Because the trinomial has a positive constant term and a negative linear term, both r and s will be negative.

Clue 2   List the factors of the quadratic term, $10x^2$, and the negative factors of the constant term, 3.

Factors of $10x^2$	Factors of 3
$x, 10x$	$-3, -1$
$2x, 5x$	$-1, -3$

Test the possibilities to see which produces $-11x$. Since $(2x - 1)(5x - 3)$ gives the correct linear term, $10x^2 - 11x + 3 = (2x - 1)(5x - 3)$.

Possible factors	Linear term
$(x - 3)(10x - 1)$	$(-1 - 30)x = -31x$
$(x - 1)(10x - 3)$	$(-3 - 10)x = -13x$
$(2x - 3)(5x - 1)$	$(-2 - 15)x = -17x$
$(2x - 1)(5x - 3)$	$(-6 - 5)x = -11x$ ←

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

1. $2x^2 + 5x + 2$
   $(2x + 1)(x + 2)$
2. $2n^2 - 7n + 3$
   $(2n - 1)(n - 3)$
3. $5y^2 - 9y - 2$
   $(5y + 1)(y - 2)$
4. $3a^2 + 7a + 2$
   $(3a + 1)(a + 2)$
5. $4y^2 - 5y + 1$
   $(4y - 1)(y - 1)$
6. $2a^2 + 11a + 5$
   $(2a + 1)(a + 5)$
7. $5a^2 - 11a + 2$
   $(5a - 1)(a - 2)$
8. $7y^2 - 9y + 2$
   $(7y - 2)(y - 1)$

5–9 Factoring Pattern for $ax^2 + bx + c$ (continued)

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

9. prime $(6k - 1)(2k - 1)$ $(4x - 3)(x + 5)$ $(2a + 5)(a + 1)$

9. $2k^2 - 5k - 1$
10. $12k^2 - 8k + 1$
11. $4x^2 + 17x - 15$
12. $2a^2 + 7a + 5$

13. $8y^2 + 6y - 9$
    $(4y - 3)(2y + 3)$
14. $9x^2 + 3x - 2$
    $(3x + 2)(3x - 1)$
15. $7k^2 - 11k - 6$
    $(7k + 3)(k - 2)$
16. $4u^2 - 8u - 5$
    $(2u + 1)(2u - 5)$

Example 3   Factor $5 - 7x - 6x^2$.

Solution
$$5 - 7x - 6x^2 = -6x^2 - 7x + 5 \quad \text{Arrange the terms by decreasing degree.}$$
$$= (-1)(6x^2 + 7x - 5) \quad \text{Factor } -1 \text{ from each term.}$$
$$= (-1)(2x - 1)(3x + 5) \quad \text{Factor the resulting trinomial.}$$
$$= -(2x - 1)(3x + 5)$$

Note: If you factor $5 - 7x - 6x^2$ directly, you will get $(5 + 3x)(1 - 2x)$. Since $(1 - 2x) = -(2x - 1)$, the two answers are equivalent.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

$-(3x - 5)(x + 2)$ $-(2x - 1)(5x + 3)$

17. $10 - 9y - 2y^2$ prime
18. $10 - x - 3x^2$
19. $3 - x - 10x^2$
20. $3 - 7x - 6x^2$
    $-(3x - 1)(2x + 3)$
21. $10 - u - 2u^2$
    $-(2u + 5)(u - 2)$
22. $5 + 8x - 4x^2$
    $-(2x - 5)(2x + 1)$

Example 4   Factor $5a^2 + 2ab - 7b^2$.

Solution
$$5a^2 + 2ab - 7b^2 = (a \quad)(5a \quad) \quad \text{Write the factors of } 5a^2.$$
$$= (a - ?)(5a + ?) \quad \text{Test possibilities.}$$
$$= (a - b)(5a + 7b)$$

Note: If you write $(a + ?)(5a - ?)$ as the second step, you will not find a combination of factors that produces the desired linear term.

Factor. Check by multiplying the factors.
23. $(x - 5y)(x + 4y)$ 24. $(2a - 3b)(2a + b)$ $(3a + 4b)(a - 3b)$

23. $x^2 - xy - 20y^2$
24. $4a^2 - 4ab - 3b^2$
25. $3a^2 - 5ab - 12b^2$
26. $5a^2 + 2ab - 7b^2$
    $(5a + 7b)(a - b)$
27. $2x^2 - xy - 3y^2$
    $(2x - 3y)(x + y)$
28. $8y^2 - 6yz - 9z^2$
    $(4y + 3z)(2y - 3z)$

Mixed Review Exercises

2. $(x - 3)(x - 4)$
5. $(3y + 11x)(3y - 11x)$
7. $(y + 9)(y + 4)$

Factor.

1. $x^2 - 196$ $(x + 14)(x - 14)$
2. $x^2 - 7x + 12$
3. $r^2 - 5r - 36$ $(r - 9)(r + 4)$
4. $c^2 - 10c + 25$ $(c - 5)^2$
5. $9y^2 - 121x^2$
6. $4a^2 - 25$ $(2a + 5)(2a - 5)$
7. $y^2 + 13y + 36$
8. $p^2 + 14p + 49$ $(p + 7)^2$
9. $9y^2 + 12y + 4$ $(3y + 2)^2$
10. $m^2 - m - 56$
    $(m - 8)(m + 7)$
11. $n^2 + 13n + 36$
    $(n + 9)(n + 4)$
12. $b^2 - 3b - 54$
    $(b - 9)(b + 6)$